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Intransitive Preferences in Retailing

by

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According to Arrow's well-known 'paradox of majority voting', intransitivities can occur in the attempt to establish a social, inter-personal preference ordering. As has been shown by May [1954], for example, a logical equivalent of this paradox can also arise in the case of individual, personal preference orderings. The present paper shows that such cases are quite plausible and natural, in as common a context as the choice of a grocery store. The paper also shows that intransitivities may occur more generally, as a result of decision rules other than the logical equivalent of majority voting.

INTRODUCTION

A shop has several different attributes relevant to choice behaviour: proximity, price level, range of goods on sale and service level. The question considered in this paper is how consumers weigh these attributes in their choice of a shop. If shop A is superior to shop B in at least one attribute, while it is not inferior in others, the choice will clearly be shop A. But what if A is superior in some respects and inferior in others? How does a consumer balance differences in distance against differences in price level or service? We will construct some plausible decision rules, and we will consider whether they satisfy the demand for transitivity of preferences that is usually found in economic theory.

DECISION RULE 1: THE BEST IN MOST RESPECTS

Perhaps the following decision rule is a plausible one: confronted with the need for a choice between any two shops, a consumer X chooses the one which 'is best in most respects'. Suppose that the attributes of a shop can be reduced to a minimum of three: proximity, cheapness and service attraction. Let shop A be represented by a vector of scores on each of these three dimensions:

$$A = (a_1, a_2, a_3)$$

where: a_1 = score on proximity
a_2 = score on cheapness
a_3 = score on service attraction

The decision rule can then be formalised as follows:

$A > B$ (A is preferred to B) if, for example:

$a_1 > b_1$, $a_2 > b_2$, $a_3 > b_3$, where $a_1 > b_1$ means that on proximity A has a higher score than B.

or: $a_1 > b_1$, $a_2 = b_2$, $a_3 = b_3$

$A \simeq B$ (indifference with respect to A and B) if, for example:

$a_1 = b_1$, $a_2 = b_2$, $a_3 = b_3$

or: $a_1 > b_1$, $a_2 < b_2$, $a_3 = b_3$, where $a_2 < b_2$ means that on cheapness A has a lower score than B.

In fact, the ordering according to service attraction is itself problematic. Service consists of several dimensions: width of assortment (number of product groups on sale), depth of assortment (range of brands, qualities and package sizes per product group), service/self-service, queuing time, friendliness of attendants, shop design and atmosphere, etc.

We will now consider the implications of the decision rule for the choice among different types of shop in the grocery trade, such as:

A: The small neighbourhood grocer, providing a fairly narrow range of product groups ('assortment width'), and an intermediate range of choice per product group ('assortment depth'), a more congenial atmosphere, and counter service. The distance to its customers is relatively short. The shop serves only a small area and hence few customers, yielding a small turnover. With the offer of an intermediate depth of assortment in spite of this, the average stockturn per item is low. The small size plus the low stockturn and the costs of service require a relatively high price level.

B: The town supermarket, providing a wide assortment with an intermediate depth, a less intimate atmosphere and limited counter service. It serves a wider area of customers, who come less frequently but buy more per visit, yielding a larger and faster turnaround even on more specialised items. The large size, faster stockturn and limited service allow for a lower price level. The average distance to its customers is clearly larger than for A.

C: The discount store, yielding a wide but shallow, ill-balanced and often changing assortment, at a cheap off-beat location, in often unattractive surroundings, without service and with little attention to display and atmosphere. The lower cost of location and other cost advantages allow for a price level that is lower than in B. The average distance to customers may be about the same as for B.

Now we order these shops according to their attributes of proximity, cheapness and service:

proximity: the neighbourhood grocer (A) will tend to be the nearest. The cash and carry store (C) may be nearer than the supermarket (B). In that case we have:

$$a_1 > c_1 > b_1$$

cheapness: the cash and carry store (C) is the cheapest and the neighbourhood store (A) the most expensive:

$$c_2 > b_2 > a_2$$

service: different consumers will evaluate the service dimension differently: some may have a preference for assortment width (one stop shopping), others for assortment depth, others for counter service (to provide help and information) and others for self-service (for speedier shopping). But let us consider a type of consumer who prefers at least a moderate attention to atmosphere, surroundings and assortment depth, and next appreciates assortment width. The ordering could then be as follows:

$$b_3 > a_3 > c_3$$

The overall result is that excellence in one attribute tends to be combined with an intermediate or bad score on the other attributes.

With the present decision rule, this ordering pattern yields the following result:

$A > C$: A is preferred to C, because its shorter distance ($a_1 > c_1$) and better service ($a_3 > c_3$) overrule its higher price ($c_2 > a_2$).

$C > B$: C is preferred to B, because its shorter distance ($c_1 > b_1$) and lower price ($c_2 > b_2$) overrule its lower service ($b_3 > c_3$).

$B > A$: B is preferred to A, because its lower price ($b_2 > a_2$) and better service ($b_3 > a_3$) overrule its greater distance ($a_1 > b_1$).

But this result violates the axiom of transitivity of preferences, according to which $A > C$ and $C > B$ should have implied $A > B$.

THE PARADOX

The intransitivities yield a paradox. Its logical structure is identical to that of Arrow's 'paradox of majority voting', but the interpretation and context are different. In Arrow's paradox we have, instead of the three attributes 1, 2 and 3, three individuals choosing from A, B and C (presumably on the basis of a single 'essential' or 'dominant' attribute). The first individual ranks A highest and B lowest, the second C highest and A lowest, and the third B highest and C lowest. The paradox arises when a social preference ordering is to be decided by majority voting (logically equivalent to the decision rule of our consumer X to prefer the shop which is better in the majority of attributes). Arrow's paradox refers to the measurability of inter-personal preference, while we are discussing personal choice.

In the literature the possibility of complex choice, i.e. single person multiple orderings, has been reported as well. Thus May [1954] conducted an experiment with college students, ranking hypothetical marriage partners on basis of the attributes intelligence, looks and wealth. Seventeen out of 62 students came up with an intransitive ('cyclical') ordering as the result of a failure to identify a 'dominating attribute', leading to the logical equivalent of the paradox of majority voting. May concluded: 'There seems no way to avoid considering intransitivity as a natural phenomenon' [May, 1954: 8]. While there is some recognition of the possible relevance of complex choice in the form of single person multiple orderings, the tendency is to transform it into the more 'familiar' problem of multi-person simple orderings. Thus, May also, having concluded that intransitivity is a 'natural phenomenon', concerns himself with the aggregation of transitive orderings 'which appears to be of the greatest interest to economists'. [May, 1954: 6]

Luce [1956: 178] also admitted that intransitive preferences are supported by experiments and personal experience, but noted that 'The axiom is not . . . easily sacrificed. It is necessary if one is to have numerical order preserving utility functions, and such functions seem indispensable for theories – such as game theory – which rest on preference orderings'. Apparently, the prime motive for denying intransitive preferences is the preservation of existing theory. Luce further referred to 'the important subjective contention that a "rational" preference ordering should satisfy the transitivity condition', and quoted Savage [1954: 21] in saying that if intransitivity of his preferences were brought to his attention he would 'feel uncomfortable in much the same way that I do when it is brought to my attention that some of my beliefs are logically contradictory'. That may be so, but can one conclude that when made aware of intransitivities, consumers will in fact revise their preferences so as to make them transitive? In any case the point remains that as a matter of fact consumers may not be aware of any intransitivities in the preferences on which they nonetheless act. Can the existence of a certain behaviour be denied on the ground that it is not logically consistent?

Luce [1956] and later papers [Blair and Pollak, 1979; Blair, 1979] accepted intransitive indifference but maintained transitive preference, but I propose that the possibility of intransitive preference should be considered, in spite of its implications for established theory. Intransitive preferences follow from complex choice on the basis of multiple attributes that do not necessarily allow for a single dominating attribute, a strict hierarchy of attributes or for projection into a single joint measure.

Complex choice is a straightforward corollary of the absence of a single and unique 'essence' even in use objects, and as May said is indeed a natural phenomenon. Therefore, we propose that as a matter of basic principle, individual preferences are at least just as problematic as social preferences, and for the same reason: both are concerned with multiple orderings. One could even argue that intransitivities are more likely to occur in personal than in interpersonal multiple orderings, since it may be more likely that of the alternatives one must choose from some are good in those respects in which others are bad, and vice versa, than that people sharing a common culture arrive at opposite preference orderings; it is such reversals of order which give rise to intransitivities.

But how do we deal with the paradox of a consumer X with cyclical preferences? She must make a choice (or she'll die of hunger), but whichever of the three alternatives she chooses, there is always another which is preferred. Thus, it seems that the choice between A, B and C is 'impossible'. Yet we cannot say that X is indifferent with respect to A, B and C, because the fact remains that she has a clear preference whenever presented with two alternatives (there is no dilemma). But she cannot make a consistent three-way choice (there is a trilemma). Paradoxical or not, we propose that it may occur, and that it does occur, and that it does not indicate irrationality on the part of the consumer. How does she make her choice? She probably randomises between the alternatives [Quirk and Saposnik, 1968: 15].

Her choice may be random (she throws dice), or it may be a stochastic process depending on some or many partly subconscious drives and stimuli. It is the business of marketing and advertising to find these out. The choice may vary, depending on whether she has the car today (giving an advantage to B) or feels like walking to the shop, looking around and having a chat (giving advantage to A) or is anxious to meet her monthly budget (giving an advantage to C). In short, she will make choices which seem natural to housewives, disloyal to (some) shopkeepers, fickle but perhaps controllable to (some) marketeers, and irrational to (some) economists.

DECISION RULE 2: NON-LINEAR DIFFERENTIAL WEIGHTS

Perhaps the paradox of cyclical preferences arises only for the decision rule of 'the best in most respects'. Perhaps that decision rule is not very plausible after all, since, in our application to the choice of shop, a small difference in distance together with a small difference in range of choice (service attraction) may overrule a large difference in price, as in the preference for the neighbourhood service grocer (A) above the more distant discount store (C). Perhaps it is this prevalence of two small differences over one large difference that causes the intransitivity.

Let us therefore adjust the decision rule to let large differences weigh more heavily than small ones. However, we maintain the assumption that consumers do not evaluate a shop singly and in absolute terms on the basis of its scores on different attributes, regardless of any alternative, but evaluate any shop in relative terms, on the basis of differences in attributes with respect to alternatives.

Suppose that in making a choice between two shops A and B a consumer considers the differences in each of the three attributes, which we now assume to be cardinally measurable; applies certain transformations to these differences (such as attaching different weights), and then adds them up to construct an overall cardinal measure of the preference for A relative to B.

For example:

$$M(A,B) = \text{sign}(a_1 - b_1)\,\alpha_1\,|a_1 - b_1|^{\beta_1} + \text{sign}(a_2 - b_2)\,\alpha_2\,|a_2 - b_2|^{\beta_2} + \text{sign}(a_3 - b_3)\,\alpha_3\,|a_3 - b_3|^{\beta_3}\ ;\ \beta_1, \beta_2 \text{ and } \beta_3 \neq 1$$

$$\text{where sign}(a_1 - b_1) = 1 \text{ if } a_1 \geqslant b_1$$
$$= -1 \text{ if } a_1 < b_1$$

It follows that $M(A,B) = -M(B,A)$.

The decision rule, called the rule of 'non-linear differential weights', now is as follows:

$$A > B \text{ if } M(A,B) > 0$$
$$A \simeq B \text{ if } M(A,B) = 0$$

With this rule let us us again consider the type of ordering pattern that we considered before, with reversals in the ordering for different attributes. As discussed, this is a pattern that is not unusual in a comparison between shops, because for a given type of shop a high score on one attribute tends to be combined with an intermediate or low score on other attributes, in comparison with a competing type of shop.

The assumed ordering pattern is as follows:

$$\begin{array}{lll} a_1 & b_2 & c_3 \\ b_1 & c_2 & a_3 \\ c_1 & a_2 & b_3 \end{array}$$

To simplify the arithmetic we assume:

$$a_1 - b_1 = b_1 - c_1 = b_2 - c_2 = c_2 - a_2 = c_3 - a_3 = a_3 - b_3 = x$$

$$\text{hence} = a_1 - c_1 = b_2 - a_2 = c_3 - b_3 = 2x$$

and we assume: $\alpha_1 = \alpha_2 = \alpha_3 = 1$
$\beta_1 = \beta_2 = \beta_3 = \beta$

Applying the decision rule we then have:

$$M(A,B) = M(B,C) = M(C,A) = 2x^{\beta} - (2x)^{\beta} = (2-2^{\beta})\,x^{\beta}.$$

if β is 1 this yields indifference between A, B and C.

if $\beta < 1$ it yields $M(A,B) > 0$, hence $A > B$
$M(B,C) > 0$, hence $B > C$
$M(C,A) > 0$, hence $C > A$

if $\beta > 1$ it yields $M(A,B) < 0$, hence $B > A$
$M(B,C) < 0$, hence $C > B$
$M(C,A) < 0$, hence $A > C$

Thus if $\beta \neq 1$, the axiom of transitivity is not satisfied.

We find the case that $\beta > 1$ rather plausible: when in some aspect the difference between two shops increases, the weight of that difference in the overall measure increases as well, so that the effect of a difference increases faster than in proportion to its size. This model thus covers the situation that some difference becomes 'exorbitant'. The same result is obtained if the increase is not that of a power function as considered here, but hyperbolic or exponential, for example.

The crux of this situation is that the consumer constructs an overall measure not on the basis of the absolute properties of each alternative (a_1, a_2, a_3), but on the basis of the differences in properties between alternatives ($a_1 - b_1$, $a_2 - b_2$, $a_3 - b_3$). This seems plausible.

It is not only the assumed ordering pattern that yields intransitivity under the present decision rule. For example, consider the following pattern:

$$a_1 > b_1 = c_1$$
$$c_2 > b_2 > a_2$$
$$a_3 > b_3 > c_3$$

Here A scores highest on two out of the three attributes, and B on none.

The relevance of this pattern can be shown as follows: A is again the neighbourhood grocer, B a supermarket and C a discount store. The first attribute is proximity, which is highest for A and the same between B and C. The second attribute is cheapness, which is highest for C, lowest for A and intermediate for B. The third attribute is service attraction. We now assume that attractive surroundings, counter service and a reasonably balanced choice of products are the dominating characteristics of service. Then A may have the highest score, C the lowest, and B an intermediate one. With this pattern, the first decision rule of 'the best in most respects' would yield: A > B, A > C, B = C. This does not violate the axiom of transitivity.

The rule of non-linear differential weights, however, may yield an intransitive preference ordering. Suppose, for example, that:

$$\alpha_1 = \alpha_2 = \alpha_3 = 1; \beta_1 = \beta_2 = \beta_3 = 2$$
$$a_1 - b_1 = a_1 - c_1 = 8, \text{ hence } b_1 - c_1 = 0$$
$$c_2 - a_2 = 10, b_2 - a_2 = 8, \text{ hence } c_2 - b_2 = 2$$
$$a_3 - b_3 = 2, a_3 - c_3 = 5, \text{ hence } b_3 - c_3 = 3$$

We now find that:

$$M(A,B) = (a_1 - b_1)^2 - (a_2 - b_2)^2 + (a_3 - b_3)^2 = 4 > 0, \text{ hence } A > B$$
$$M(B,C) = (b_1 - c_1)^2 - (c_2 - b_2)^2 + (b_3 - c_3)^2 = 5 > 0, \text{ hence } B > C$$
$$M(C,A) = (a_1 - c_1)^2 - (c_2 - a_2)^2 + (a_3 - c_3)^2 = 11 > 0, \text{ hence } C > A$$

This preference ordering is intransitive. We can also find values for the differences in attributes which yield a transitive preference ordering. The more general solution is worked out in an appendix.

CONCLUSION

One could probably find many more examples of intransitive preferences, with varying degrees of plausibility with respect to the decision rules, and varying degrees of likelihood of the patterns of multiple ordering. Summing up, we propose that intransitive personal preferences are not only a logical possibility, but actually are quite plausible and likely to occur, in the choice between shops at least.

Some people may now say that intransitive preferences do arise but that they are then 'naive', in the sense that people, when confronted with the paradox involved, will still take a decision, so that intransitivity does not 'really' occur. But this confuses the existence of a paradox with its resolution. It is not good enough to say that the fact that decisions are made 'proves' that preferences are transitive, for the following reasons:

- If it were valid, one might just as well say that the fact that social decisions are made disproves Arrow's paradox of majority voting.
- The argument in fact states not that transitive orderings are an empirical fact but that they constitute a logical necessity. Then there is no difference in content between the statement 'people make decisions' and 'preferences are transitive', because they are then tautological, and the concept of 'preference' has lost all meaning.

The claim of transitivity should be based on empirical arguments. The existence of a preference should be inferred not from a single decision but from decisions systematically going in a certain direction. Otherwise we would not be able to distinguish between preference and indifference either, because in the case of indifference, decisions are made as well. A more or less random choice behaviour may then indicate either a situation of indifference or a paradox of circular preference. This is observed in the lack of any systematic tendency in the selection between alternatives. Next, one might say that the analysis is superfluous, then, since from an operational point of view there is no difference between the hypothesis of transitivity and intransitivity: simply equate circular preference with indifference. But this is not valid. Let us compare the situation of indifference between A, B and C ($A \simeq B \simeq C$) and circular preference ($A > B > C > A$). If any of the three shops, say C, goes out of business or is replaced by a new one which is worse in all respects, say, than A and B, then the intransitive pattern will predict a clear preference for, and a systematic choice of, A above B, while according to the indifference pattern the consumer will still toss a coin between A and B.

APPENDIX: REGIONS OF INTRANSITIVE PREFERENCE

We consider the following multiple ordering:

(1) $p_1 > r_1 > q_1$
$q_2 > p_2 = r_2$
$q_3 > r_3 > p_3$
and the decision rule

(2) $P > Q$ if $M(P, Q) > 0$; $P = Q$ if $M(P, Q) = 0$ where:

(3) $M(P, Q) = (p_1 - q_1)^2 - (q_2 - p_2)^2 - (q_3 - p_3)^2$
$M(Q, R) = -(r_1 - q_1)^2 + (q_2 - r_2)^2 + (q_3 - r_3)^2$
$M(R, P) = -(p_1 - r_1)^2 + (r_3 - p_3)^2$

The question is for what values of the differences in the three attributes between P, Q and R we will find the intransitive ordering $P > Q > R > P$

This will arise if:

(4) $M(P, Q) > 0$; $M(Q, R) > 0$; $M(R, P) > 0$

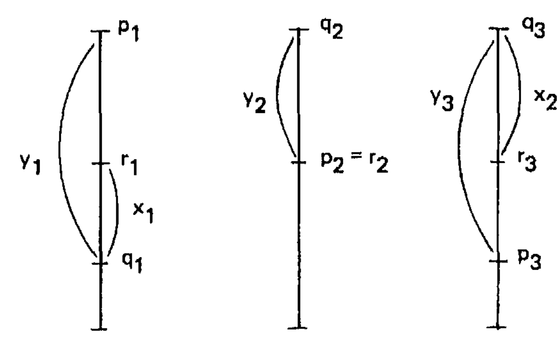
We define:

$y_1 = p_1 - q_1, y_2 = q_2 - p_2 = q_2 - r_2, y_3 = q_3 - p_3, x_1 = r_1 - q_1, x_2 = q_3 - r_3$

Hence: $p_1 - r_1 = y_1 - x_1$ and $r_3 - p_3 = y_3 - x_2$

This is illustrated in Figure 1.

FIGURE 1



Now (3) and (4) yield:

(5) (5.1) $M(P, Q) = y_1^2 - y_2^2 - y_3^2 > 0$
(5.2) $M(Q, R) = -x_1^2 + y_2^2 + x_2^2 > 0$
(5.3) $M(R, P) = -(y_1 - x_1)^2 + (y_3 - x_2)^2 > 0$

To preserve the ordering (1) we further require:

(6) (6.1) $x_1 < y_1$ and $x_2 < y_3$
(6.2) y_1, y_2, y_3, x_1 and x_2 are all non-negative.

We note that intransitivity will not occur only under the conditions (4), but also if:

(7) $M(P, Q) < 0$ hence $Q > P$
$M(Q, R) < 0$ hence $R > Q$
$M(R, P) < 0$ hence $P > R$

To find the corresponding 'feasible regions' we reverse all the inequality signs in (5). For the ordering pattern of Figure 1 we then obtain intransitivities if, given a value for y_3, we select values for y_1, y_2, x_1 and x_2 either from the areas A_1 and A_2, or from the areas B_1 and B_2 in Figures 2 and 3.

We could consider the intransitive regions for other patterns of multiple ordering, but it is not our current purpose to give an exhaustive treatment. We noted that the region of intransitivity in the case considered here is restricted. This does not by itself give a measure of the likelihood of intransitivity given the ordering pattern. To arrive at such a measure we would first have to specify a probability density function across the y_1, y_2 and x_1, x_2 planes.

FIGURE 2

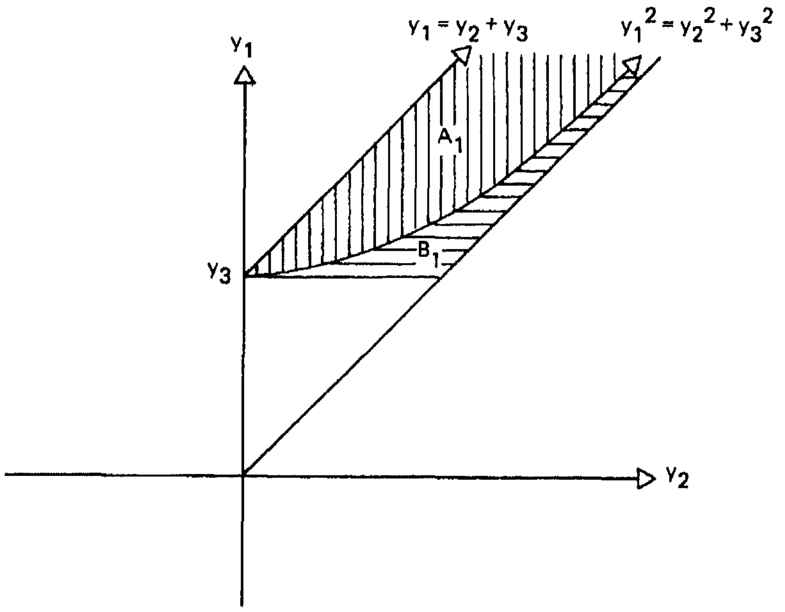
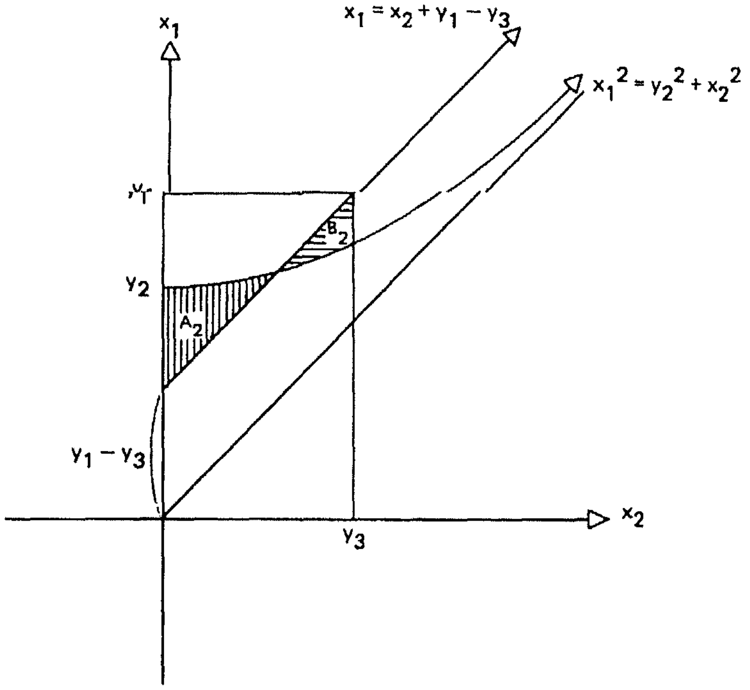


FIGURE 3



For a completely general treatment of intransitivity we would have to include:

- all possible patterns of multiple ordering;
- for each pattern the intransitive regions together with probability density functions;
- alternative weighting functions (apart from the quadratic one considered here);
- ordering patterns with more than three attributes.

We will not attempt such a general treatment here.

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